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# Nonlinear Response of Vortex Breakdown over a Pitching Delta Wing

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**A flow visualization study was carried out to investigate the response of breakdown location to small amplitude pitching oscillations of a delta wing. At moderate angles of attack, left and right breakdown locations are in-phase and locked to the pitching frequency. At large angles of attack, low-frequency antisymmetric modes and the symmetric mode at the pitching frequency are observed simultaneously. The existence of multiple frequencies in the spectra and other features suggest that vortex breakdown behaves like a nonlinear self-excited oscillator.**

## Nomenclature

$c$ = chord length
$f$ = frequency
$f_0$ = natural frequency of the antisymmetric mode
$R$ = ratio defined in Eq. (2)
$Re$ = Reynolds number
$r$ = correlation coefficient
$S$ = power spectral density
$t$ = time
$U_\infty$ = freestream velocity
$x$ = streamwise distance of breakdown location from the apex of the wing
$\bar{x}$ = time-averaged breakdown location
$x'$ = fluctuations of breakdown location
$x_{\rm rms}$ = rms value of breakdown location
$\alpha$ = angle of attack
$\alpha_0$ = mean angle of attack
$\alpha_1$ = amplitude of pitching motion
$\Delta t$ = sampling time interval
$\Lambda$ = sweep angle

*Subscripts*

left = left vortex breakdown
right = right vortex breakdown
rms = root-mean-square value

## Introduction

IT has been shown previously that the vortex breakdown location over stationary delta wings is not steady and exhibits fluctuations in the streamwise direction[1–4] (Fig. 1). The amplitude of these fluctuations can be a significant fraction of the chord length.[1] These fluctuations, as well as the unsteady nature of flow downstream of vortex breakdown,[5] cause buffeting and poor control.[6] The vortex breakdown phenomenon and the aerodynamics of stationary and unsteady delta wings at high angle of attack have been reviewed in several articles.[7–10]

Experiments for several delta wings of varying aspect ratios showed that a peak in the spectrum of breakdown location existed. It was shown that the peak observed in the spectrum of fluctuations of breakdown location was a result of the antisymmetric motion of breakdown locations for left and right vortices, which suggested an interaction between these vortices.[3,4] This antisymmetric motion can be demonstrated by studying the difference between the breakdown locations $(x_{\rm left} - x_{\rm right})/c$ and the average breakdown location $(x_{\rm left} + x_{\rm right})/2c$. The spectra of these are shown in Fig. 2 for $\Lambda = 75$ deg and $\alpha = 42$ deg (adapted from Ref. 4). It is seen that most of the energy is concentrated in the difference, and there is a dominant peak corresponding to the quasiperiodic antisymmetric motion. It was shown that the most coherent oscillations occurred for the largest angle of attack or sweep angle, when the breakdown locations were closest to each other. Similar observations of the quasiperiodic oscillations of breakdown location over stationary delta wings were also made by others[11–14] in different facilities. The frequency of this organized motion, given by the preceding references, is shown in Table 1. When compared with the frequencies of the other instabilities (helical mode instability, Kelvin–Helmholtz instability, and vortex shedding), the frequency range of the oscillations of breakdown location for a stationary delta wing is much closer to the frequency range of typical aerodynamic maneuvers (up to $fc/U_\infty \cong 0.03$) for conventional fighter aircraft.

If each breakdown is considered to be an oscillator, one can imagine that we have two coupled oscillators, similar to two mechanical oscillators coupled by means of a spring. If the mechanical oscillators start with equal but opposite displacements, the subsequent motion of each oscillator will be harmonic with a certain frequency in the absence of damping. This normal mode of motion is antisymmetric in the sense that the displacements are equal, but opposite. This mechanical system can oscillate in harmonic motion at this frequency without a driving force. Although it does not do justice to the complexity of the fluid dynamics, the self-excited quasiperiodic motion of breakdown locations can be viewed similar to that of coupled oscillators.

The main objective of this work is to investigate the effect of dynamic wing motion on the fluctuations of breakdown location. The response of breakdown location and possible coupling between the wing motion and breakdown location is very important. In particular, there exists a possibility of a coupling of this instability with the pitching motion of the wing. In this study, we investigated the response of breakdown location to external forcing in the form of small-amplitude pitching oscillations. This response is expected to be a mode competition between the natural antisymmetric oscillations and the symmetric oscillations caused by the pitching motion of the wing. A flow visualization study was conducted to investigate the unsteady response of breakdown locations.

## Experimental Facility

Flow visualization was performed in a water channel with a cross-sectional area of 61 × 61 cm. The turbulence level in the channel was 0.6%. The delta wing model had a sweep angle of $\Lambda = 75$ deg, and a chord length of $c = 203$ mm. The lee surface was flat, whereas the leading edges were beveled at 45 deg on the windward side. A

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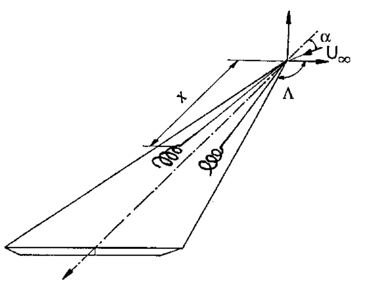
**Table 1 Observations of the quasiperiodic oscillations of breakdown locations over stationary delta wings**

| Reference | $\Lambda$, deg | Dimensionless frequency, $fc/U_\infty$ |
|---|---|---|
| Ayoub and McLachlan[11] | 76 | 0.10–0.17 |
| Portnoy[12] | 60, 65, 70, 75 | 0.04–0.10 |
| Helin and Watry[13] | 60 | 0.10 |
| Gursul and Yang[2] | 70 | 0.06–0.12 |
| Johari et al.[14] | 60 | —— |
| Menke et al.[3] | 65, 70, 75 | 0.04–0.12 |



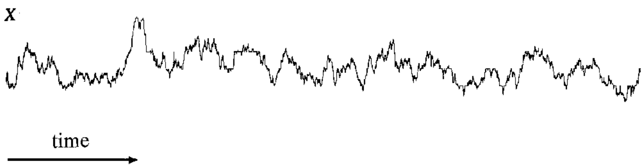




**Fig. 1 Streamwise location of vortex breakdown as a function of time for $\alpha$ = 37 deg, $\Lambda$ = 70 deg. Length of time record is ~ $100c/U_\infty$.[2]**

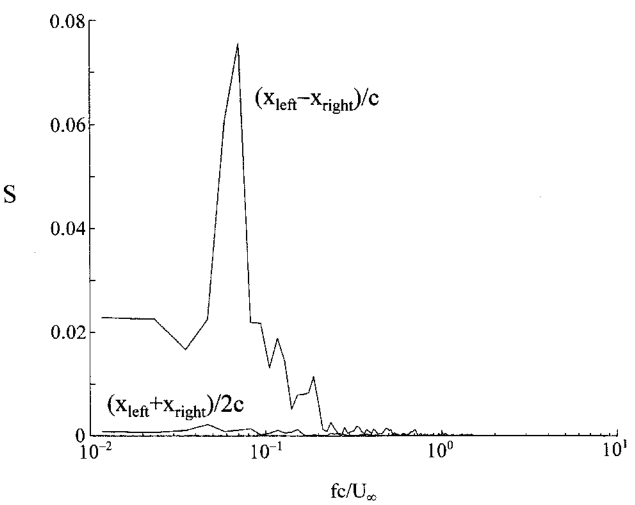



**Fig. 2 Spectra of difference and average of breakdown locations, $\alpha$ = 42 deg, and $\Lambda$ = 75 deg.**

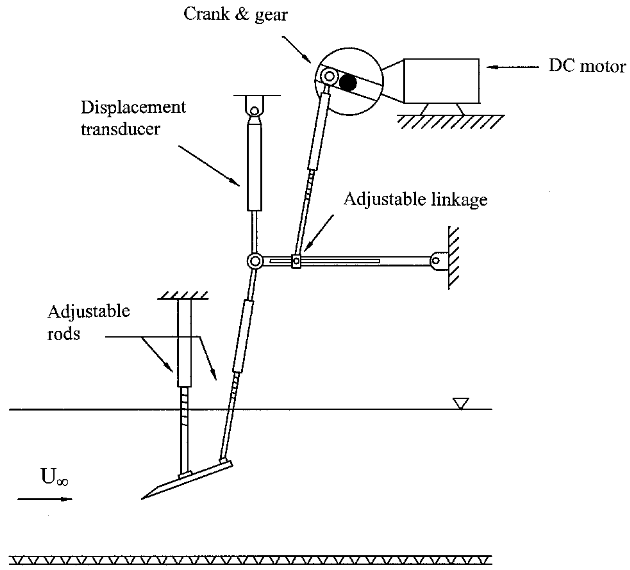



**Fig. 3 Schematic of the experimental setup.**

schematic of the experimental setup is shown in Fig. 3. The model was supported with two vertical struts to avoid any interference with the leading-edge vortices. The Reynolds number based on the chord length was $Re = 4.1 \times 10^4$.

Flow visualization of vortex breakdown was performed by injecting fluid with food-coloring dye near the apex of the model. The motion of vortex breakdown location was recorded by a video system, which consisted of a VCR (at 30 frames/s resolution), a frame counter/window inserter, and a charge-coupled device camera with a zoom lens. The videotape recording of the motion was analyzed frame-by-frame, and the chordwise distance of breakdown location from the apex was measured. The breakdown was spiral-type most of the time, as generally observed over delta wings. However, the bubble-type breakdown could be observed intermittently. The measurement uncertainty for the breakdown location was $0.005c$. The time history of the vortex breakdown location was obtained for a total time record of about $(135\text{–}170)c/U_\infty$. For the experimental conditions, the smallest time resolution is limited by the frame speed, and corresponds to $0.033c/U_\infty$. Therefore, the frequencies up to $fc/U_\infty = 15$ can be resolved in the frequency domain. Analyzing the flow visualization results frame-by-frame proved to be very time consuming (more than 4000 frames needed to be studied if each frame was analyzed). Because of this reason, larger time steps were used in analyzing some cases, where a very small time step was not necessary.

The pitching mechanism was similar to the one used by LeMay et al.[15] It was described in detail by Menke.[16] A displacement transducer was used to monitor the angle of attack variation. A variable-speed dc motor and speed controller were used to drive the pitching mechanism.

## Results

To investigate the response of breakdown location to small-amplitude external forcing, the delta wing was pitched about the midchord at several values of amplitude and frequency:

$$\alpha = \alpha_0 + \alpha_1 \cos(2\pi f t) \tag{1}$$

where the amplitude was varied as $\alpha_1 = 1$, 2, and 3 deg, and the dimensionless frequency was varied as $fc/U_\infty = 0.1$, 0.5, and 1.0. (The smallest pitching frequency was limited by the experimental setup, and was around $fc/U_\infty = 0.1$.) The mean angle of attack $\alpha_0$ was varied between 29 and 42 deg. The response of breakdown location mainly depends on $\alpha_0$. In Fig. 4, the time histories of left and right breakdowns (solid and dashed lines) are shown for $\alpha_0 = 29$ and 42 deg, for $\alpha_1 = 3$ deg, and $fc/U_\infty = 0.1$. The breakdown locations are in-phase for $\alpha_0 = 29$ deg, and locked to the forcing frequency. On the other hand, for $\alpha_0 = 42$ deg, the breakdown locations are not locked to the forcing frequency, and there is a remarkable antisymmetry between the left and right breakdowns. For stationary delta wings, the natural antisymmetric mode is observed for all angles of attack when vortex breakdown is over the wing.[3] However, the amplitude of the quasiperiodic oscillations increases as the angle of attack is increased, because the left and right breakdowns get closer to each other. For a pitching wing, the symmetric mode may completely suppress the antisymmetric mode when the breakdowns are near the trailing edge, as shown in Fig. 4 for $\alpha_0 = 29$ deg. For other mean angles of attack between 29 and 42 deg, a trend of increasing antisymmetry was found as the angle of attack increases.[16] Because

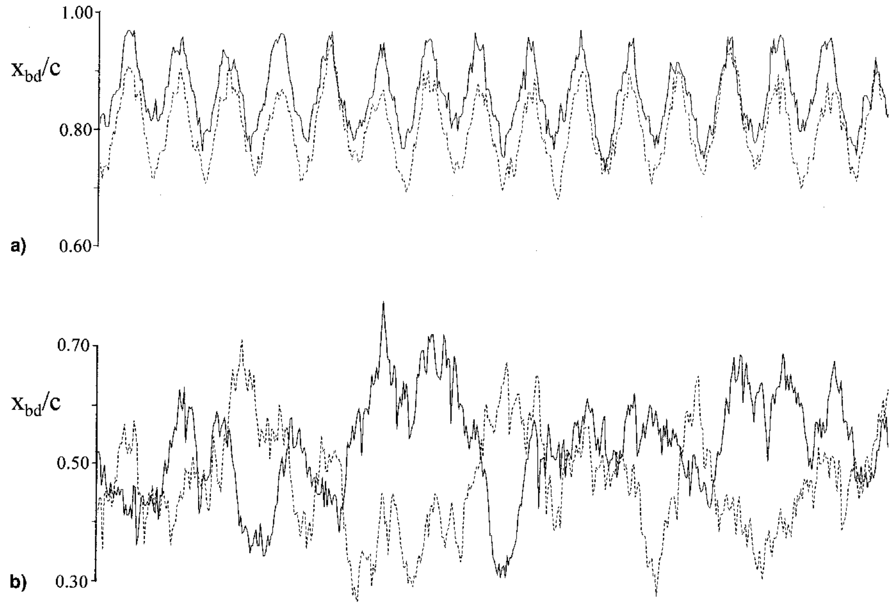



**Fig. 4 Time histories of breakdown locations for left and right breakdowns for a) $\alpha_0$ = 29 deg and b) $\alpha_0$ = 42 deg, $\alpha_1$ = 3 deg, and $fc/U_\infty$ = 0.1. Length of time record $170c/U_\infty$, and sampling time interval $\Delta t = 0.333c/U_\infty$.**

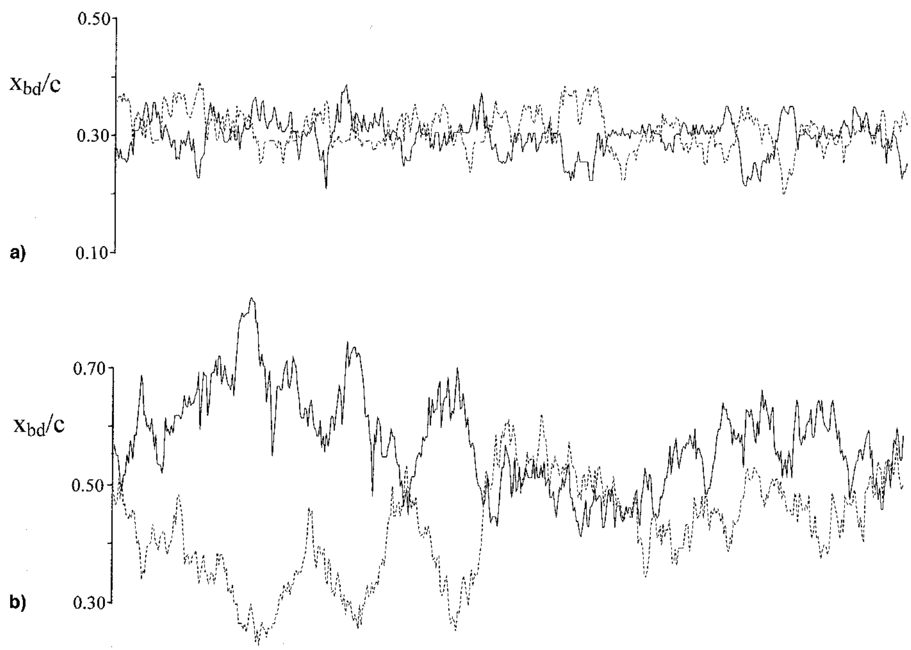



**Fig. 5 Time histories of breakdown location for left and right breakdowns for a) stationary wing and b) pitching wing, $\alpha_0$ = 42 deg, $\alpha_1$ = 1 deg, and $fc/U_\infty$ = 0.1. Length of time record $170c/U_\infty$, and sampling time interval $\Delta t = 0.333c/U_\infty$.**

of its interesting features, a parametric study was conducted for $\alpha_0 = 42$ deg. It was generally observed that the amplitude of the fluctuations of breakdown location was relatively large. In fact, this type of response was obtained even for the smallest amplitude of pitching motion $\alpha_1 = 1$ deg. In Fig. 5, the time histories of breakdown locations are shown for $\alpha_0 = 42$ deg, $\alpha_1 = 1$ deg, and $fc/U_\infty = 0.1$. For comparison, the time histories of breakdown locations for the stationary wing at $\alpha_0 = 42$ deg are also shown. The amplitude of the oscillations for the pitching wing is much larger. For the pitching wing, the large-amplitude low-frequency oscillations are evident, and the antisymmetry persists almost all of the time.

Because the difference and average of the left and right breakdown locations give an indication of antisymmetric and symmetric modes, respectively, $(x_{\text{left}} - x_{\text{right}})/c$ and $(x_{\text{left}} + x_{\text{right}})/2c$ were studied for each case. The spectra of these quantities for $\alpha_0 = 42$ deg, $\alpha_1 = 3$ deg, and $fc/U_\infty = 0.1$ are shown in Fig. 6. The spectrum of the difference shows two dominant frequencies: $fc/U_\infty \approx 0.03$ and $fc/U_\infty \approx 0.06$. The larger of the two frequencies is the natural frequency that is also observed for the stationary wing (see Fig. 2). The smaller frequency, $fc/U_\infty \approx 0.03$, may be the subharmonic of the natural frequency. By comparing with Fig. 2, it is concluded that oscillations are amplified for pitching wing at the natural frequency. In other words, the pitching motion increases the amplitude of the antisymmetric motion of breakdown locations. The spectrum of the average shows the forcing (pitching) frequency ($fc/U_\infty = 0.1$). It is clear that the antisymmetric mode is dominant in this case. The spectra for $\alpha_1 = 1$ deg are similar (not shown here), but the amplitude of the forcing frequency becomes even smaller, whereas the amplitude of the natural frequency becomes very large compared with that of the stationary wing.

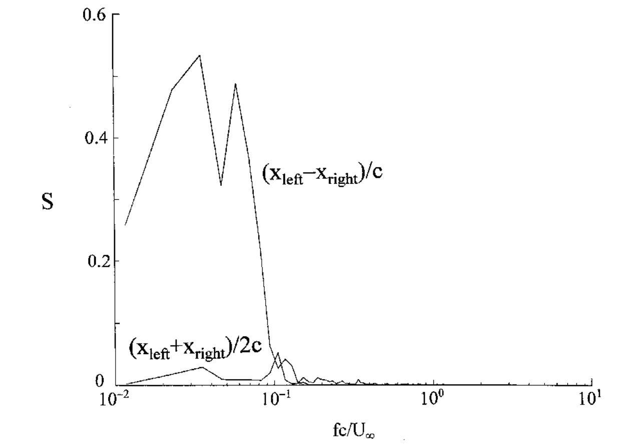



Fig. 6 Spectra of difference and average of breakdown for $\alpha_0$ = 42 deg, $\alpha_1$ = 3 deg, and $fc/U_\infty$ = 0.1.

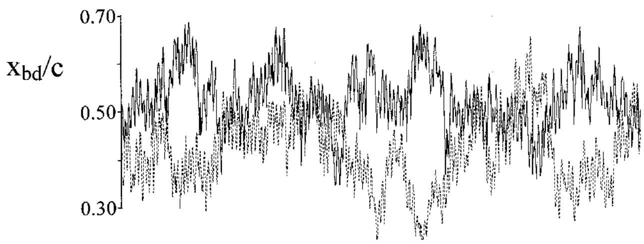



Fig. 7 Time histories of breakdown location for left and right breakdowns for $\alpha_0$ = 42 deg, $\alpha_1$ = 3 deg, and $fc/U_\infty$ = 1.0. Length of time record 135 $c/U_\infty$, and sampling time interval $\Delta t$ = 0.033$c/U_\infty$.

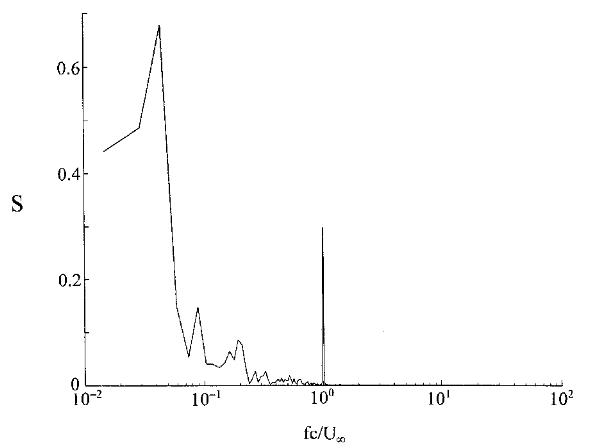



Fig. 8 Spectrum of breakdown location for $\alpha_0$ = 42 deg, $\alpha_1$ = 3 deg, and $fc/U_\infty$ = 1.0.

The response of breakdown location to external forcing as shown in these cases can be understood if it is considered to be a nonlinear system that has self-sustained oscillations with a certain frequency $f_0$. If an external periodic excitation with $f$ is applied, in some cases, the forcing extinguishes the original oscillation with $f_0$, and the system oscillates with $f$. In several other cases, the effects may be the opposite: the system begins to oscillate with its own natural frequency, as in Fig. 6. This phenomenon is called asynchronous excitation by Minorsky.[17] This phenomenon is possible only when the system is self-excited. Moreover, this type of response was only observed if the oscillator was of a hard type, which required some impulse to start the oscillations.

In Fig. 7, the time histories of breakdown locations are shown for $\alpha_0 = 42$ deg, $\alpha_1 = 3$ deg, and $fc/U_\infty = 1.0$. Again, a low frequency as well as the forcing frequency components are observed. The antisymmetry at the low frequency is remarkable. The spectrum of the left breakdown location is shown in Fig. 8. The spectral amplitude

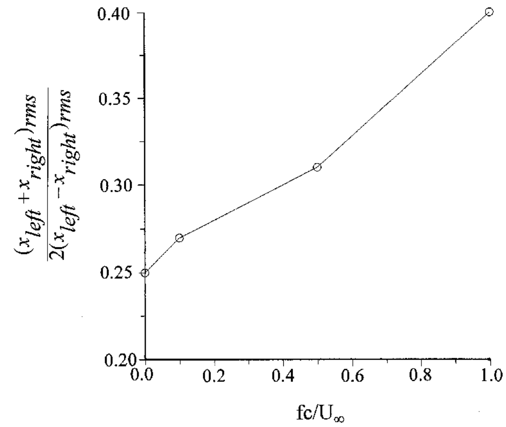



Fig. 9 Variation of the ratio $R$ with reduced frequency, $\alpha_0$ = 42 deg and $\alpha_1$ = 3 deg.

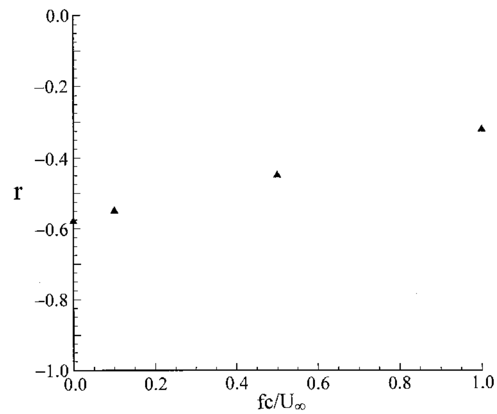



Fig. 10 Variation of the correlation coefficient $r$ with reduced frequency, $\alpha_0$ = 42 deg and $\alpha_1$ = 3 deg.

at the forcing frequency $fc/U_\infty = 1.0$ (symmetric mode) is on the same order as the amplitude at the low frequency $fc/U_\infty \approx 0.04$ (antisymmetric mode). In fact, relative amplitude of the symmetric mode increases with the increasing reduced frequency. This can be shown by considering the rms values of the difference and the average. The ratio of these rms values is defined as

$$R = \frac{(x_{\text{left}} + x_{\text{right}})_{\text{rms}}}{2(x_{\text{left}} - x_{\text{right}})_{\text{rms}}} \tag{2}$$

The variation of this ratio with reduced frequency is shown in Fig. 9. It is seen that this ratio increases with increasing reduced frequency, confirming that the symmetric mode becomes more important with increasing reduced frequency. The effect of the reduced frequency on the mode competition can be also seen by studying the correlation coefficient defined as

$$r = \frac{\overline{x'_{\text{left}} x'_{\text{right}}}}{(x'_{\text{left}})_{\text{rms}} (x'_{\text{right}})_{\text{rms}}} \tag{3}$$

The variation of the correlation coefficient with the reduced frequency is shown in Fig. 10. It is seen that the correlation coefficient increases with reduced frequency because of the increasing symmetry. Figures 9 and 10 demonstrate that the relative amplitude of the symmetric mode increase with the increasing pitching frequency. The details of the time-dependent flowfield are not known to be able to explain this observation. We can only speculate that the coupling between the breakdowns is not as effective when the forcing frequency is much larger than the natural frequency.

The rms of breakdown location for left and right vortices as a function of the forcing frequency is shown in Fig. 11 for $\alpha_1 = 3$ deg

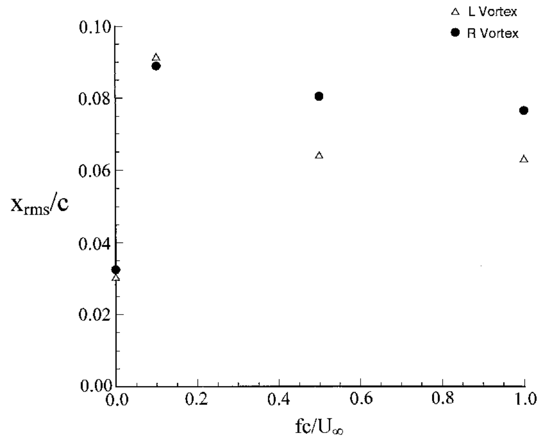



**Fig. 11 Root-mean-square value of fluctuations of breakdown location as a function of forcing frequency, $\alpha_0$ = 42 deg and $\alpha_1$ = 3 deg.**

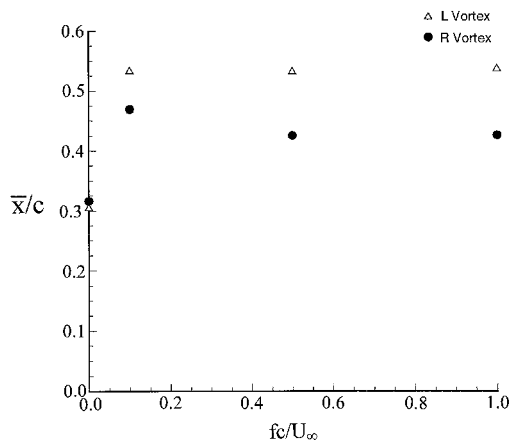



**Fig. 12 Time-averaged breakdown location as a function of forcing frequency, $\alpha_0$ = 42 deg and $\alpha_1$ = 3 deg.**

and $\alpha_0 = 42$ deg. The amplitude of the fluctuations becomes maximum around the natural frequency. Note that the rms values of left and right breakdowns are nearly the same for the stationary wing and for $fc/U_\infty = 0.1$. However, the rms values become different at larger frequencies. In Fig. 12, the time-averaged breakdown location is shown as a function of the forcing frequency. Although the average breakdown location is symmetric for the stationary delta wing, an asymmetry in terms of the time-averaged breakdown location develops for the pitching delta wing. The time-averaged breakdown location also appears to move downstream with forcing.

## Conclusions

A flow visualization study was carried out to investigate the effect of small-amplitude pitching motion on the fluctuations of breakdown location. It is shown that the response of breakdown location mainly depends on the $\alpha_0$. Left and right breakdown locations are in-phase and locked to the forcing frequency for moderate mean angles of attack. On the other hand, at a large angle of attack, multiple frequencies are observed in the spectra. Low-frequency antisymmetric modes and the symmetric mode at the pitching frequency are observed simultaneously. The low-frequency modes are believed to be related to the natural antisymmetric mode of the stationary wing (with some frequency modulation), and are the dominant mode at high angles of attack. The relative amplitude of the symmetric mode increases with reduced frequency, which is also confirmed by calculating the correlation coefficient. The pitching motion also introduces some asymmetries in terms of the time-averaged breakdown locations.

The response of breakdown location to small-amplitude pitching oscillations of the delta wing showed that it has some features of a nonlinear system, such as the existence of multiple frequencies in the spectrum. Also, there are other indications that it has some similarities to a self-excited oscillator. Even very small-amplitude pitching motion is sufficient to cause a large increase in the amplitude of the antisymmetric oscillations of breakdown location compared with that of the stationary wing.

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